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Flow separation on a β-plane

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Boundary-layer structure of prograde and retrograde rotating flows past a cylinder on a β-plane is investigated. It is found that β inhibits boundary-layer separation for prograde flows but it exerts no influence on the boundary-layer structure for retrograde flows. The results agree with the few available experimental observations.

1. Introduction

In an experimental study of slightly viscous prograde (eastward) flow past a right circular cylinder in a rotating system on a β-plane, White (1971) obtained a qualitative agreement with theoretical results based on inviscid considerations. The experimental evidence presented is rather limited but it seems that flow separation is inhibited although the Reynolds number based on the radius of the cylinder is 90.† In non-rotating systems separation occurs for $Re \simeq 2{\cdot}2$ (Coutanceau & Bouard 1977). Recently Merkine & Solan (1979) determined the boundary-layer structure of flow past a cylinder in a rotating system on an f-plane. Their analysis indicates that rotation inhibits separation only if the spin-down of vorticity induced by the horizontal Ekman layers is comparable to the advection of vorticity in the vertical boundary layer along the cylinder. The flow remains fully attached provided $E_v^{\frac{1}{2}}/(2^{\frac{1}{2}}Ro) > 1$, where Ro and E_v are the appropriate Rossby and Ekman numbers defined in §2. If $E_v^{\frac{1}{2}}/Ro \ll 1$ the inhibiting effect of the horizontal Ekman layers disappears and the vorticity dynamics in the vertical boundary layer is identical to that of the classical non-rotating case regardless of the smallness of the Rossby number.‡ In White's experiments $E_v^{\frac{1}{2}}/(2^{\frac{1}{2}}Ro) \simeq 0{\cdot}15$ yet separation seems to be inhibited. These experiments are characterized by the presence of the β-effect which accounts for the variability of the Coriolis parameter. It introduces a new $O(\beta)$ vorticity source which influences the boundary-layer dynamics and consequently the phenomenon of flow separation.

It is the purpose of the following analysis to investigate the influence of β on the flow separation problem. We restrict ourselves to the parameter space for which $E_v^{\frac{1}{2}}/Ro \ll 1$ such that the influence of the secondary circulation induced by the Ekman layers can be neglected and the problem can be treated as two dimensional.

† The kinematic viscosity is not specified in the experiments. We assumed it to be $0{\cdot}01\ \mathrm{cm^2\,s^{-1}}$ as appropriate to water at temperature of 20 °C.

‡ The calculations of Merkine & Solan (1979) were repeated as a result of a recently discovered syntax error in the computer program. Slight quantitative changes which did not affect the condition for flow separation were found. The new drawings will be supplied by the authors upon request.

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2. Formulation

We consider a steady homogeneous flow past a right circular cylinder of radius R with generators parallel to the axis of rotation. The cylinder extends throughout the depth of the fluid. The z axis of a right-handed Cartesian co-ordinate system coincides with the axis of the cylinder and the y axis points poleward. The dependence of the Coriolis parameter, f, on latitude is introduced through the β-plane approximation, i.e. $f = f_0 + \beta' y$. The flow, which is unbounded laterally, approaches the cylinder with a uniform prograde or retrograde velocity U, i.e. in the positive-x (eastward) or negative-x (westward) direction respectively.

For reasons discussed earlier the flow field can be considered horizontal and consequently it is governed by the two-dimensional vorticity equation, which in non-dimensional form can be written using polar co-ordinates as

$$u_r \frac{\partial}{\partial r}(\zeta + \beta r \sin\theta) + u_\theta \frac{1}{r}\frac{\partial}{\partial \theta}(\zeta + \beta r \sin\theta) = \frac{1}{Re}\nabla^2 \zeta, \tag{1}$$

$$\zeta = \frac{1}{r}\frac{\partial (r u_\theta)}{\partial r} - \frac{1}{r}\frac{\partial u_r}{\partial \theta}; \tag{2}$$

u_r and u_θ are the radial and azimuthal components of velocity respectively and ζ is the vertical component of the relative vorticity. The continuity equation

$$\frac{1}{r}\frac{\partial (r u_r)}{\partial r} + \frac{1}{r}\frac{\partial u_\theta}{\partial \theta} = 0 \tag{3}$$

can be satisfied by introducing the stream function ϕ such that

$$u_r = -\frac{1}{r}\frac{\partial \phi}{\partial \theta}, \quad u_\theta = \frac{\partial \phi}{\partial r}, \tag{4}$$

$$\zeta = \nabla^2 \phi = \frac{1}{r}\frac{\partial}{\partial r}\left(r \frac{\partial \phi}{\partial r}\right) + \frac{1}{r^2}\frac{\partial^2 \phi}{\partial \theta^2}. \tag{5}$$

The radius of the cylinder R and the upstream velocity U were chosen as the relevant scales; $Re = UR/\nu$ is the Reynolds number of the flow. It is the ratio of the Rossby number $Ro = U/(f_0 R)$ to the horizontal Ekman number $E_H = \nu/(f_0 R^2)$. The no-slip boundary condition at the wall requires that

$$u_r = u_\theta = 0 \quad \text{on} \quad r = 1. \tag{6}$$

Away from the cylinder the upstream velocity should be approached, i.e.

$$(u_r, u_\theta) \to \pm\ (\cos\theta, -\sin\theta), \tag{7}$$

where the + and − correspond to prograde and retrograde flow conditions respectively. We restrict ourselves to slightly viscous flows such that viscous effects are confined to thin boundary layers ahead of any point of separation. The flow exterior to the boundary layer can be considered as strictly inviscid and for a prograde upstream velocity it assumes a wavelike character. Consequently, upstream it must satisfy the radiation condition

$$r^{\frac{1}{2}}(\phi + r\sin\theta) \to 0. \tag{8}$$

The problem as posed is formally strictly two-dimensional and as such places no restriction on the smallness of the Rossby number. However, it can only be the proper limit of a flow configuration bounded vertically by horizontal planes provided it is understood that $Ro \ll 1$ and $E_v = \nu/(f_0 H^2) \ll 1$, where H is the depth of the system, and $E_v^{\frac{1}{2}}/Ro \ll 1$. We require that $Re \gg 1$. Geophysical applications require that $\beta \leqslant O(1)$, where $\beta = \beta' R^2/U$.

3. The exterior solution

Exterior to the boundary layer the right-hand side of (1) can be dropped, which implies that

$$\nabla^2\phi + \beta r \sin\theta = F(\phi) \tag{9}$$

and, in the absence of closed streamlines, we obtain that

$$\nabla^2\phi + \beta(r\sin\theta \pm \phi) = 0, \tag{10}$$

where + and − correspond to prograde and retrograde flow conditions respectively The perturbation stream function ψ is governed by

$$\nabla^2\psi \pm \beta\psi = 0, \tag{11}$$

where

$$\psi = \phi \pm r\sin\theta. \tag{12}$$

The inviscid boundary condition on the cylinder is $u_r = 0$, which implies that

$$\psi = a + \sin\theta \quad \text{on} \quad r = 1, \tag{13}$$

where a is an arbitrary constant arising from the fact that the flow domain is multiply connected. In the limit of $\beta \to 0$ the non-uniqueness is related to an arbitrary r independent circulation that can be added to the solution. When $\beta \neq 0$ such a circulation is r dependent. We demand symmetry about the x axis and consequently set a equal to zero. Such an assumption seems appropriate for an upstream uniform flow which started from rest and provided that vortex shedding arising from possible flow separation does not alter the symmetry assumption. Away from the cylinder we require

$$r^{\frac{1}{2}}\psi \to 0 \quad \text{upstream for prograde flow;} \tag{14}$$

$$\psi \to 0 \quad \text{as} \quad r \to \infty \quad \text{for retrograde flow.} \tag{15}$$

The solution for the inviscid prograde flow problem has been found by Miles (1968) in the context of lee waves generated by stratified flow past a semi-circular obstacle. Adapting Miles' solution, we have

$$\left.\begin{aligned}
\psi &= \sum_{q=1}^{\infty} g_q \psi_q, \\
\psi_q &= a_q\Big(Y_q(\beta^{\frac{1}{2}}r)\sin q\theta + \sum_{p=1}^{\infty} b_{qp} J_p(\beta^{\frac{1}{2}}r)\sin p\theta\Big), \\
a_q &= -\pi(\tfrac{1}{2}\beta^{\frac{1}{2}})^q/(q-1)!, \\
b_{qp} &= \frac{4}{\pi}\,\frac{q}{-q^2+p^2} \quad (q \text{ even}, p \text{ odd}), \\
&= \frac{4}{\pi}\,\frac{p}{-q^2+p^2} \quad (q \text{ odd}, p \text{ even}), \\
&= 0 \quad (q-p \text{ even}), \\
\sum_{q=1}^{\infty} a_q&[\delta_{pq} Y_p(\beta^{\frac{1}{2}}) + b_{qp} J_p(\beta^{\frac{1}{2}})]\, g_q = \delta_{1p}, \quad p = 1, 2, \ldots
\end{aligned}\right\} \tag{16}$$

The solution of the boundary-layer equations requires evaluation of the inviscid tangential velocity along the cylinder. We obtain

$$u_\Theta(r=1) = -\sin\theta + \sum_{q=1}^{\infty} g_q \frac{\partial\psi_q}{\partial r}(r=1)$$

$$\frac{\partial\psi_q}{\partial r}(r=1) = \tfrac{1}{2}\beta^{\frac{1}{2}} a_q((Y_{q-1}(\beta^{\frac{1}{2}}) - Y_{q+1}(\beta^{\frac{1}{2}}))\sin q\theta + \sum_{p=1}^{\infty} b_{qp}(J_{p-1}(\beta^{\frac{1}{2}}) - J_{p+1}(\beta^{\frac{1}{2}}))\sin p\theta. \quad (17)$$

The coefficients g_q given in (16) are determined by solving an infinite set of linear equations. Approximate solutions are obtained by truncation. We have retained only the first three Fourier components in the expansion for ψ, which is adequate for the range $\beta < 4$ (Miles 1968). It should be pointed out that when $\beta^{\frac{1}{2}} \simeq 1{\cdot}27$ closed streamlines appear and this violates the assumption leading to (10). The flow field contained within the closed streamlines is spun down because of the presence of Ekman layers (Ingersoll 1969) which although ignored in our analysis will eventually exert their influence on the closed circulation. For the range of parameters considered here the spin-down time is much longer than the advection time scale and the inviscid exterior solution as well as the boundary layer along the cylinder are established long before the closed circulation of the exterior solution is affected by viscosity. Both boundary-layer structure and flow separation will suffer some modification when the exterior closed circulation is spun down but this will take place only for times comparable to the spin-down time.

The inviscid solution for the retrograde flow dows not possess wavelike character and therefore it is obtained with little effort:

$$\psi = -K_1(\beta^{\frac{1}{2}}r)\sin\theta/K_1(\beta^{\frac{1}{2}}), \quad u_\theta(r=1) = [1+\tfrac{1}{2}\beta^{\frac{1}{2}}(K_0(\beta^{\frac{1}{2}})+K_1(\beta^{\frac{1}{2}}))]\sin\theta, \quad (18)$$

where K_0 and K_1 are modified Bessel functions of the second kind of orders zero and one respectively.

4. The boundary-layer structure

We consider first prograde flow. The vorticity equation (1) indicates that relative vorticity can be generated either by advection of fluid columns across constant y lines or at solid boundaries. In the boundary layer the shear vorticity dominates both curvature vorticity and the β-effect ($\beta \leqslant O(1)$) such that the appropriate vorticity balance near the cylinder is

$$u\frac{\partial^2 u}{\partial s\,\partial\eta} + v\frac{\partial^2 u}{\partial\eta^2} - \frac{\partial^3 u}{\partial\eta^3} = \max O(1/Re^{\frac{1}{2}}, \beta/Re^{\frac{1}{2}}) \quad (19)$$

as more careful analysis demonstrates. s and η are co-ordinates measured along and normal to the wall respectively with s increasing in the streamwise direction. In particular $s = \pi-\theta$, $\eta = (r-1)\,Re^{\frac{1}{2}}$, $u = -u_\theta$ and $v = Re^{\frac{1}{2}}\,u_r$. The continuity equation becomes

$$\frac{\partial u}{\partial s} + \frac{\partial v}{\partial\eta} = O(1/Re^{\frac{1}{2}}). \quad (20)$$

The relevant boundary conditions are

$$u = v = 0 \quad \text{on} \quad \eta = 0, \quad (21)$$

$$u \to U_\infty(s) \quad \text{as} \quad \eta \to \infty \quad (22)$$

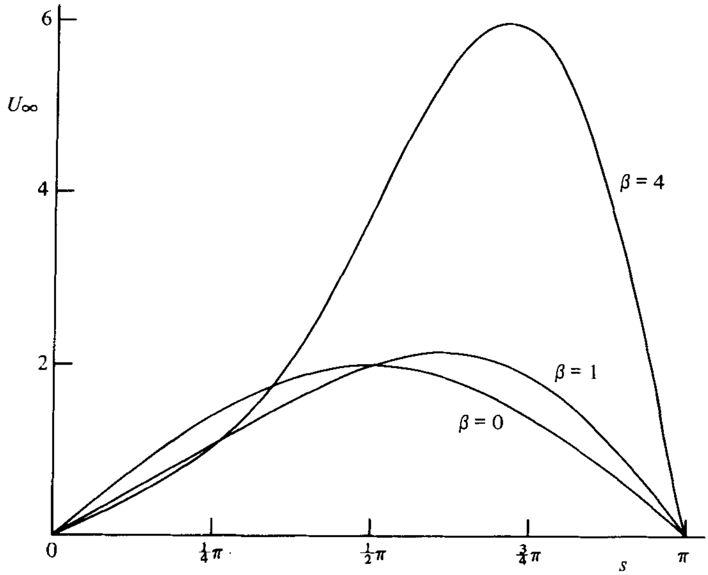

FIGURE 1. Asymptotic tangential velocity along the cylinder for prograde flows.

with $U_\infty(s)$ obtained from (17). Equation (19) can be integrated with respect to η to yield

$$u\frac{\partial u}{\partial s}+v\frac{\partial u}{\partial \eta}-\frac{\partial^2 u}{\partial \eta^2}-U_\infty\frac{dU_\infty}{ds} = \max O(1/Re^{\frac{1}{2}},\ \beta/Re^{\frac{1}{2}}). \tag{23}$$

The outcome of this analysis is that the dynamics of the boundary-layer flow is identical with that of the classical non-rotating case where the influence of β is felt only through the asymptotic boundary condition (22). The analysis for retrograde flow follows along identical lines; the only difference is that $U_\infty(s)$ is now determined by (18).

5. Results and discussion

The discussion of the last paragraph implies that the β-control of the boundary-layer structure is essentially kinematic. It is the divergence of the asymptotic tangential velocity which affects the advection of vorticity generated at the wall and consequently the wall shear stress.

For prograde flow adequate approximate expressions for the asymptotic tangential velocity are

$$\left.\begin{aligned} U_\infty(s) &= 2\sin s, \quad \beta = 0,\\ U_\infty(s) &= 2{\cdot}0372\sin s - 0{\cdot}4073\sin 2s + 0{\cdot}0357\sin 3s, \quad \beta = 1,\\ U_\infty(s) &= 4{\cdot}3258\sin s - 2{\cdot}4326\sin 2s + 0{\cdot}5592\sin 3s, \quad \beta = 4.\end{aligned}\right\} \tag{24}$$

These expressions are plotted in figure 1, which tells us that an increase in β shifts downstream the tendency for $\partial v/\partial \eta > 0$ and consequently it delays positive transverse advection of vorticity from the wall. In other words, larger β suggests thinner boundary layers. Note, however, that this is only a $\beta^{\frac{1}{2}}$-effect as implied by (17).

The pressure gradient $-U_\infty dU_\infty/ds$ appearing in (23) is explicitly a linear function of β but the dependence on β is actually stronger since higher harmonics become more

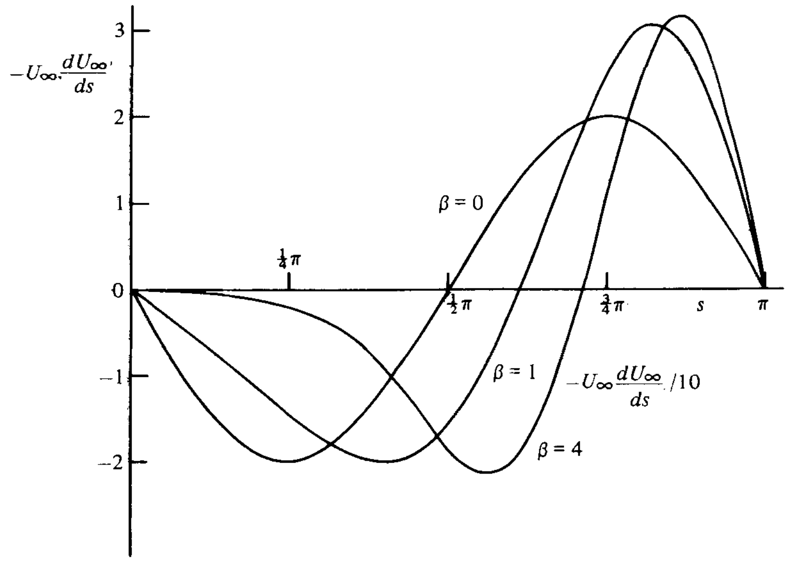

FIGURE 2. Boundary-layer pressure gradient for prograde flows.

important as β increases. This behaviour is depicted in figure 2, which shows the strong pressure gradients that develop for large β. The existence of an adverse pressure gradient is necessary for the occurrence of flow separation. Figure 2 indicates that the region of adverse pressure gradient shifts toward the rear stagnation point as β increases but for moderate β the dependence is $O(\beta^{\frac{1}{2}})$ only. (The sign change of the pressure gradient is determined by dU_∞/ds and this is related to $\partial v/\partial \eta$ and hence to the transverse vorticity advection discussed earlier.) Note that the strong pressure gradients and the positive transverse vorticity advection that develop for large β suggest that the point of separation shifts closer to the point where $dU_\infty/ds = 0$ as β increases.

The boundary-layer equations were solved using well-established techniques (Cebeci, Smith & Wang 1969). The dependence on β of the shear stress along the cylinder is shown in figure 3. With the aid of figure 1 it can be inferred that near the forward stagnation point β slightly decreases the shear stress along the wall. This is a consequence of a weakened negative transverse advection of vorticity. Downstream the trend is reversed, the shear stress increases and separation is delayed.

In contrast to the case treated by Merkine & Solan (1979) the complicated β-dependence of the asymptotic tangential velocity prohibits deriving a simple necessary condition for flow separation. Thus in order to determine the value of β necessary for the existence of a fully attached flow, assuming that separation can be prohibited for sufficiently large values of β, the problem has to be solved for successively larger values of β. We have not pursued this course for several reasons. For large values of β the exterior inviscid solution becomes less realistic with the appearance of vigorous closed circulations, not to mention the large number of terms necessary to calculate the asymptotic tangential velocity. Furthermore, for a given Reynolds number the boundary-layer approximation deteriorates with an increasing error of $O(\beta/Re^{\frac{1}{2}})$. Finally geophysical applications are limited to $\beta \leqslant O(1)$.

Profiles of the transverse velocity v are shown in figures 4 (a, b). In the outer region

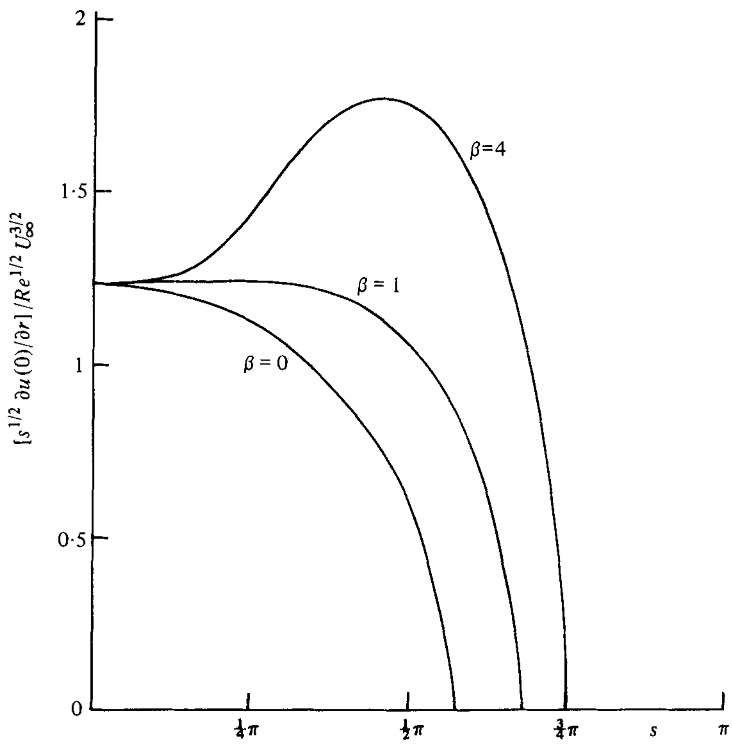

FIGURE 3. Shear stress along the cylinder for prograde flows.

of the boundary layer the transverse dependence of v is consistent with the streamwise dependence of the asymptotic tangential velocity through the continuity equation. This is also true throughout the boundary layer in regions where the divergence of $U_\infty(s)$ is not small. However, near the stationary point of U_∞ the inner region of the boundary layer is dominated by the viscosity-induced streamline displacement effect and v is positive (see v at 120° for $\beta = 4$).

Representative profiles of the tangential velocity at various stations along the cylinder are shown in figure 5. All profiles coincide at the forward stagnation point, i.e. $s = 0$ (see also figure 3) but evolve differently in the streamwise direction. Consistently with figure 3 the inflexion point appears first for smaller values of β.

We discuss now the solution for retrograde flows. Equation (18) tells us that the asymptotic tangential velocity has the same functional form as for the non-rotating case ($\beta = 0$) but with a different multiplicative constant. (For example, $U_\infty = 2{\cdot}7 \sin s$ for $\beta = 1$ in contrast to $U_\infty = 2 \sin s$ for $\beta = 0$. For $\beta > O(1)$ an inviscid boundary layer develops next to the cylinder but this is of no consequence for our discussion.) It follows that renormalization can reduce the boundary-layer equations to the case of $\beta = 0$ with a separation angle identical with that of non-rotating flows. We conclude that the β-effect does not inhibit separation for retrograde flows.

The experimental evidence available for comparison with the analysis presented in this work is very limited and difficult to interpret. White's (1971) experimental set-up is the closest to our model; however, results are presented only for prograde flow with $\beta = 4$. There is a qualitative agreement with the inviscid theory and consistently

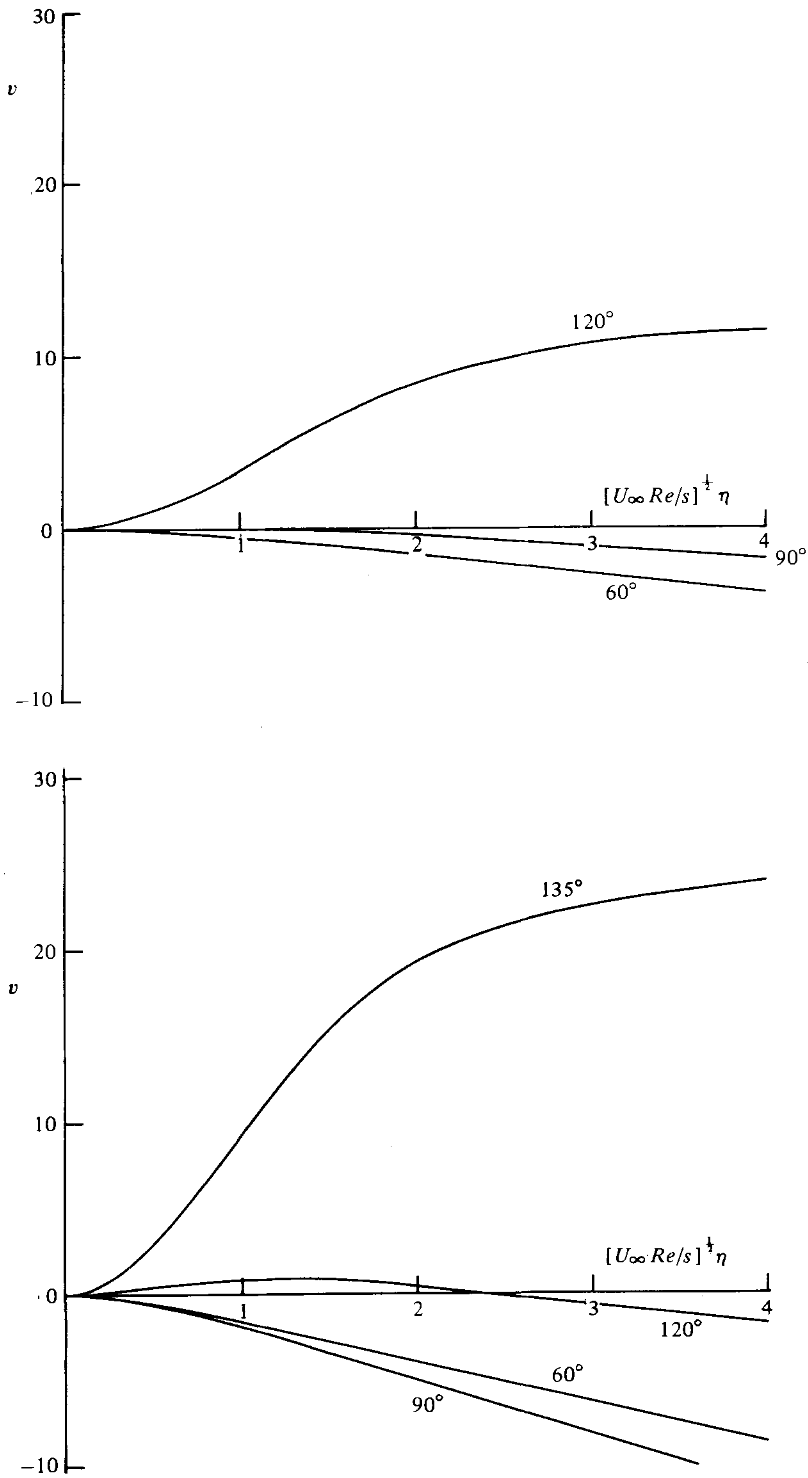

FIGURE 4. Transverse velocity profiles at various stations along the cylinder for prograde flows. (a) $\beta = 1$; (b) $\beta = 4$.

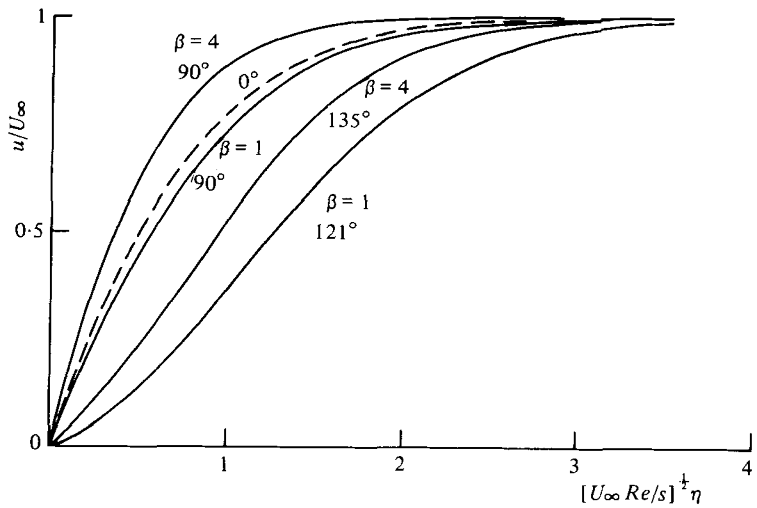

FIGURE 5. Tangential velocity profiles at various stations along the cylinder for prograde flows.

with our results flow separation does not seem to occur for angles smaller than $\frac{3}{4}\pi$ measured from the forward stagnation point. It is impossible to determine, however, whether the flow remains fully attached or that separation occurs and small-scale eddies form near the rear stagnation point.

In the early Fifties experimental investigations of rotating flows past obstacles of various shapes were performed by Fultz & Long (1951), Long (1952), Fultz and Frenzen (1955) and Frenzen (1955). The basic apparatus was a hemispherical shell, within which obstacles of various shapes could be towed along latitude circles. Some of the experiments were concerned with obstacles that completely filled the annulus gap. It is difficult to draw conclusions even from those experimental results which are pertinent to our analysis since both β and the ambient flow conditions changed considerably over the latitudinal extent of the obstacle. Nevertheless, the flow-separation-inhibiting effect of β for prograde flows can be inferred from Frenzen (1955). When the obstacle was placed sufficiently to the south a steady wave pattern was observed in the lee. As the obstacle was moved northward the wavy structure weakened and travelling cyclonic eddies were periodically shed from the obstacle. This is in accord with our results since beta, the gradient of the earth's vorticity, is maximal at the equator and zero at the north pole. Thus flow separation is more likely to occur for obstacles which are placed further to the north, i.e. for smaller values of β.

The experiments performed for retrograde flows show a different response. The flow field is no longer wavy in character, but this seems to be the only agreement with the inviscid theory. It looks as if the wake of the obstacle is dominated by viscous effects. Both Long (1952) and McCartney (1975) report that the latitudinal belt traversed by tall obstacles is blocked. This can be attributed to the circular geometry where the viscous wake extends all the way around the annulus to form a part of the oncoming flow. In a frame of reference moving with the obstacle eddying motion is observed (Fultz & Long 1951; Long 1952), and the investigators report that the edges of the blocked latitudinal belt are unsteady and that large-scale vortices form downstream of the obstacle. McCartney (1975) suggests shear-layer instability of the edges of the wake as a cause for this usteadiness but it could well be a regular vortex shedding from the cylinder.

The analysis that has been presented above suggests that β inhibits boundary-layer separation for prograde flows but it exerts no influence on the boundary-layer structure when the flows are retrograde. The limited experimental evidence available tends to support this conclusion. Our analysis can also be extended, with not much effort, to include the effect of lateral walls. The problem is, however, considerably more difficult when $E_v^{\frac{1}{2}}/Ro = O(1)$ for the exterior flow is strongly affected by vorticity spin-down and the exterior solution is inherently nonlinear.

It should be emphasized that our investigation is restricted to a cylinder that extends throughout the depth of the system. If the cylinder occupies only a fraction of the depth then β and the ratio of the fractional height of the cylinder to Ro determine whether or not an inertial Taylor column exists above the cylinder (McCartney 1975). The exterior solution for a short cylinder with an inertial Taylor column attached to it is broadly similar to the case of a cylinder that extends throughout the depth of the system (McCartney 1975) but the boundary-layer structure existing along the edges of the Taylor column could very well be different to the boundary-layer structure considered here. Nevertheless, rotation exhibits a strong constraint for two-dimensionality in rapidly rotating systems and it seems possible that a Taylor column might shed vortices if the conditions for boundary-layer separation along the truncated cylinder and inertial Taylor formation occur simultaneously. The experimental apparatus described by Vaziri (1977) is most suitable for checking this conjecture experimentally. Support for such hypothesis is provided by Takematsu & Kita (1978) who observed vortex shedding from a Taylor column in a retrograde flow where the β-effect was simulated by the slope of the parabolic free surface. A thorough survey of the various downstream effects in rotating systems can be found in a recent review by Baines & Davies (1979).

Finally, we would like to comment that the geophysical relevance of the boundary-layer analysis and the corresponding laboratory experiments referred to earlier is probably restricted to oceanography only. With $\beta' \simeq 1{\cdot}6 \times 10^{-11}\,\mathrm{m^{-1}\,s^{-1}}$ corresponding to mid-latitude conditions and typical velocities of $0{\cdot}1\ \mathrm{m\ s^{-1}}$ for the ocean and $10\,\mathrm{m\,s^{-1}}$ for the atmosphere we find that $\beta = O(1)$ for oceanic length scales of $O(100\ \mathrm{km})$ and atmospheric length scales of $O(1000\ \mathrm{km})$.

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